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Single Camera Structure and Motion

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Abstract—A reduced order nonlinear observer is proposed for the problem of "structure and motion (SaM)" estimation of a stationary object observed by a moving calibrated camera. In comparison to existing work which requires some knowledge of the Euclidean geometry of an observed object or full knowledge of the camera motion, the developed reduced order observer only requires one camera linear velocity and corresponding acceleration to asymptotically identify the Euclidean coordinates of the feature points attached to an object (with proper scale reconstruction) and the remaining camera velocities. The unknown linear velocities are assumed to be generated using a model with unknown parameters. The unknown angular velocities are determined from a robust estimator which uses a standard Homography decomposition algorithm applied to tracked feature points. A Lyapunov analysis is provided to prove the observer asymptotically estimates the unknown states under a persistency of excitation condition.

Index Terms—Motion from structure (MfS), persistency of excitation (PE), structure and motion (SaM), structure from motion (SfM).

I. INTRODUCTION

The objective of the classic "structure from motion (SfM)" problem is to estimate the Euclidean coordinates of tracked feature points attached to an object (i.e., 3-D structure) provided the relative motion between the camera and the object is known. The converse of the SfM problem is the "motion from structure (MfS) problem where the relative motion between the camera and the object is estimated based on known geometry of the tracked feature points attached to an object. An extended problem proposed in this technical note is "structure and motion (SaM) where the objective is to estimate the Euclidean geometry of the tracked feature points as well as the relative motion between the camera and tracked feature points. The SaM problem is a fundamental problem and some examples indicate that SaM estimation is only possible up to a scale when a pinhole camera model is used. To recover the scale information, either the linear camera velocities or partial information about the structure of the object, e.g., a known length between two feature points in a scene is required (cf. [1]–[3]). In this technical note,

a partial solution to the SaM problem is presented, where the objective is to asymptotically identify the Euclidean 3-D coordinates of tracked feature points and the camera motion, provided at least one linear velocity of the camera is known. The angular velocity is estimated using a filter. The estimated angular velocity and a measured linear velocity are combined to estimate the scaled 3-D coordinates of the feature points.

Solutions to the SfM, MfS, and the SaM problems can be broadly classified as offline methods (batch methods) and online methods (iterative methods). References and critiques of batch methods can be found in [4], [5] and the references therein. Batch methods extract an image data set from a given image sequence and then the 3-D structure is reconstructed from the data set. These methods are usually based on nonlinear optimization, projective methods, or invariant-based methods. Often batch methods lack an analytical analysis of convergence, with the exception of results such as [6], [7] using convex optimization techniques. The main drawback of batch methods is that they cannot be used to execute online/real-time tasks. Thus, the need arises for iterative or online methods with analytical guarantees of convergence.

Online methods typically formulate the SfM and MfS problems as a continuous differential equation, where the image dynamics are derived from a continuous image sequence (see [1], [3], [8]–[12] and the references therein). Online methods often rely on the use of an Extended Kalman filter (EKF) [1], [13]. Kalman filter based approaches also lack a convergence guarantee and could converge to wrong solutions in practical scenarios. Also, *a priori* knowledge about the noise is required for such solutions. In comparison to Kalman filter-based approaches, some researchers have developed nonlinear observers for SfM with analytical proofs of stability. For example, a discontinuous high-gain observer called identifier-based observer (IBO) is presented for structure estimation in [12] under the assumption of known camera motion. In [8], a discontinuous sliding-mode observer is developed which guarantees exponential convergence of the states to an arbitrarily small neighborhood, i.e., uniformly ultimately bounded (UUB) result. A continuous observer which guarantees asymptotic structure estimation is presented in [9] under the assumption of known camera motion. An asymptotically stable reduced-order observer is presented in [14] to estimate the structure given known camera motion. Under the assumption that a known Euclidean distance between two feature points is known, a nonlinear observer is used in [3] to asymptotically identify the camera motion. In contrast to these approaches, the method in this technical note does not assume any model knowledge of the structure and only requires one linear velocity and the corresponding acceleration.

Various batch and iterative methods have been developed to solve the SaM problem upto a scale, such as [15], [16]. However, in comparison to SfM and MfS results, sparse literature is available where the SaM problem is formulated in terms of continuous image dynamics with associated analytical stability analysis. In [17], an algorithm is presented to estimate the structure and motion parameters up to a scaling factor. In [18], a perspective realization theory for the estimation of the shape and motion of a moving planar object observed using a static camera up to a scale is discussed. Recently, a nonlinear observer is developed in [10] to asymptotically identify the structure given the camera motion (i.e., the SfM problem) or to asymptotically identify the structure and the unknown time-varying angular velocities given all three linear velocities. In [1], [2] structure and linear velocities are estimated given partial structure information such as length between two points on an object, which may be difficult in practice for random objects. In another recent result in [19], the IBO approach in [12] is used to estimate the structure and the constant angular velocity of the camera given all three linear velocities, which may not be possible in practical scenarios such as a camera attached to a unmanned vehicle where sideslip

velocities may not be available. The problem of estimating structure, time varying angular velocities, and time varying linear velocities of the camera without knowledge of partial structure information remains an unsolved problem.

The technical challenge presented by the SaM problem is that the image dynamics are scaled by an unknown factor, and the unknown structure is multiplied by unknown motion parameters. As described in the following sections, the challenge is to estimate a state in the open loop dynamics that appears nonlinearly inside a matrix that is multiplied by a vector of unknown linear and angular velocity terms [see (6)]. By assuming that the velocities are known, or some model knowledge exists, previous online efforts have been able to avoid the problem of separately estimating multiplicative uncertainties. The contribution of this work is a strategy to segregate the multiplicative uncertainties, and then to develop a reduced order nonlinear observer to address the SaM problem where the structure (i.e., the properly scaled relative Euclidean coordinates of tracked feature points), the time-varying angular velocities, and two unknown time-varying linear velocities are estimated (i.e., one relative linear velocity is assumed to be known along with a corresponding acceleration). The result exploits an uncertain locally Lipschitz model of the unknown linear velocities of the camera. The strategic use of a standard Homography decomposition is used to estimate the angular velocities, provided the intrinsic camera calibration parameters are known and feature points can be tracked between images. A persistency of excitation (PE) condition is formulated, which provides an observability condition that can be physically interpreted as the known camera linear velocity should not be zero over any small interval of time, and the camera should not be moving along the projected ray of a point being tracked. A Lyapunov-based analysis is provided that indicates the SaM observer errors are globally asymptotically regulated provided the PE condition is satisfied. By developing a reduced order observer to segregate and estimate the multiplicative uncertainties, new applications can be addressed including: range and velocity estimation using a camera fixed to a moving vehicle where only the forward velocity/acceleration of the vehicle is known.

II. EUCLIDEAN AND IMAGE SPACE RELATIONSHIPS

Consider a moving camera that views four or more planar[1] and noncollinear feature points (denoted by $j = \{1, 2, \ldots, n\} \; \forall n \geq 4$) lying fixed in a visible plane π_r, attached to an object in front of the camera. Let $\mathcal{F}_r$ be a static coordinate frame attached to the object. A static reference orthogonal coordinate frame $\mathcal{F}_c^*$ is attached to the camera at the location corresponding to an initial point in time t_0 where the object is in the camera field of view (FOV). After the initial time, an orthogonal coordinate frame $\mathcal{F}_c$ attached to the camera undergoes some rotation $\bar{R}(t) \in SO(3)$ and translation $\bar{x}_f(t) \in \mathbb{R}^3$ away from $\mathcal{F}_c^*$.

The Euclidean coordinates $\bar{m}_j(t) \in \mathbb{R}^3$ of the feature points expressed in the current camera frame $\mathcal{F}_c$ and the respective normalized Euclidean coordinates $m_j(t) \in \mathbb{R}^3$ are defined as

$$\bar{m}_j(t) = \begin{bmatrix} x_{1j}(t) & x_{2j}(t) & x_{3j}(t) \end{bmatrix}^T,$$
$$m_j(t) = \begin{bmatrix} \dfrac{x_{1j}(t)}{x_{3j}(t)} & \dfrac{x_{2j}(t)}{x_{3j}(t)} & 1 \end{bmatrix}^T. \tag{1}$$

The constant Euclidean coordinates and the normalized coordinates of the feature points expressed in the camera frame $\mathcal{F}_c^*$ are denoted by $\bar{m}_j^* \in \mathbb{R}^3$, and $m_j^* \in \mathbb{R}^3$, respectively, and are given by (1) superscripted by a "$*$." Consider a closed and bounded set

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[1]Four planar points are needed to compute the homography. The homography can also be computed with eight non-coplanar and non-collinear feature points using the "virtual parallax" algorithm [20].

$\mathcal{Y} \subset \mathbb{R}^3$. Auxiliary state vectors $y_j^* = [y_{1j}^*, y_{2j}^*, y_{3j}^*]^T \in \mathcal{Y}$ and $y_j(t) = [y_{1j}(t), y_{2j}(t), y_{3j}(t)]^T \in \mathcal{Y}$ are constructed from (1) as

$$y_j^* = \begin{bmatrix} \dfrac{x_{1j}^*}{x_{3j}^*} & \dfrac{x_{2j}^*}{x_{3j}^*} & \dfrac{1}{x_{3j}^*} \end{bmatrix}^T,$$
$$y_j = \begin{bmatrix} \dfrac{x_{1j}}{x_{3j}} & \dfrac{x_{2j}}{x_{3j}} & \dfrac{1}{x_{3j}} \end{bmatrix}^T. \tag{2}$$

The corresponding feature points m_j^* and $m_j(t)$ viewed by the camera from two different locations (and two different instances in time) are related by a depth ratio $\alpha_j(t) \triangleq x_{3j}^*/x_{3j}(t) \in \mathbb{R}$ and a Homography matrix $H(t) \in \mathbb{R}^{3 \times 3}$ as[2]

$$m_j = \alpha_j H m_j^*. \tag{3}$$

Using projective geometry, the normalized Euclidean coordinates m_j^* and $m_j(t)$ can be related to the pixel coordinates in the image space as

$$p_j = A m_j, \quad p_j^* = A m_j^* \tag{4}$$

where $p_j(t) = [u_j, v_j, 1]^T$ is a vector of the image-space feature point coordinates $u_j(t), v_j(t) \in \mathbb{R}$ defined on the closed and bounded set $\mathcal{I} \subset \mathbb{R}^3$, and $A \in \mathbb{R}^{3 \times 3}$ is a constant, known, invertible intrinsic camera calibration matrix [22]. Since A is known, (4) can be used to recover $m_j(t)$, which can be used to partially reconstruct the state $y(t)$.

Assumption 1: The relative Euclidean distance $x_{3j}(t)$ between the camera and the feature points observed on the target is upper and lower bounded by some known positive constants (i.e., the object remains within some finite distance away from the camera).

Remark 1: The definition in (2) along with Assumption 1 can be used to conclude that $y_3(t)$ remains in a set Ω defined as $\Omega = \{y_3 \mid \underline{y}_3 \le y_3 \le \bar{y}_3\}$ where $\bar{y}_3, \underline{y}_3 \in \mathbb{R}$ denote known positive bounding constants. Likewise, since the image coordinates are constrained (i.e., the target remains in the camera field of view) the relationships in (1), (2), and (4) along with the fact that A is invertible can be used to conclude that $\bar{y}_1 \ge |y_{1j}(t)| \ge \underline{y}_1$, $y \ge |y_{2j}(t)| \ge \underline{y}_2$ where $\bar{y}_1, \bar{y}_2, \underline{y}_1, \underline{y}_2 \in \mathbb{R}$ denote known positive bounding constants[3]. Furthermore, the velocity parameters of the camera are bounded by constants.

III. PERSPECTIVE CAMERA MOTION MODEL

At some spatiotemporal instant, the camera views a point q on the object. The point q can be expressed in the coordinate system $\mathcal{F}_c$ as

$$\bar{m} = x_f + R x_{o'q} \tag{5}$$

where $x_{o'q}$ is a vector from the origin of the static coordinate system F_r, attached to the object at the point q. By differentiating (5), the relative motion of q can be expressed as [22], [23]

$$\dot{\bar{m}} = [\omega]_\times \bar{m} + b$$

where $\bar{m}(t)$ is defined in (1), $[\omega]_\times \in \mathbb{R}^{3 \times 3}$ denotes a skew symmetric matrix formed from the angular velocity vector of the camera $\omega(t) = [\omega_1, \omega_2, \omega_3]^T \in \mathcal{W}$, and $b(t) = [b_1, b_2, b_3]^T \in \mathcal{B}$ denotes the linear velocity of the camera. The sets $\mathcal{W}$ and $\mathcal{B}$ are closed and bounded sets

such that $\mathcal{W} \subset \mathbb{R}^3$ and $\mathcal{B} \subset \mathbb{R}^3$. Using (2) and (5), the dynamics of the partially measurable state $y(t)$ can be expressed as

$$\begin{aligned}
\dot{y}_1 &= (b_1 - y_1 b_3) y_3 - y_1 y_2 \omega_1 + (1 + y_1^2)\omega_2 - y_2 \omega_3 \\
\dot{y}_2 &= (b_2 - y_2 b_3) y_3 - (1 + y_2^2)\omega_1 + y_1 y_2 \omega_2 + y_1 \omega_3 \\
\dot{y}_3 &= -y_3^2 b_3 - y_2 y_3 \omega_1 + y_1 y_3 \omega_2
\end{aligned} \tag{6}$$

where $y_1(t)$ and $y_2(t)$ can be measured through the transformation in (4).

IV. STRUCTURE AND MOTION ESTIMATION

A. Estimation With a Known Linear Velocity

In this section, an estimator is designed for the perspective dynamic system in (6), where the angular velocity is considered unknown and only one of the linear velocities (i.e., b_3)[4] and respective acceleration (i.e., $\dot{b}_3$) is available. Moreover, an uncertain dynamic model of the linear velocity $b(t)$ is assumed to be available as [8], [10]

$$\dot{b}_i(t) = q(b_i, t) b_i, \forall i = \{1, 2\} \tag{7}$$

where $q(b_i, t) \in \mathbb{R}$ is a known locally Lipschitz function of unknown states.

To facilitate the design and analysis of the subsequent observer, a new state $u(t) \in \mathcal{U} \subset \mathbb{R}^2 \triangleq [u_1(t) = y_3 b_1, u_2(t) = y_3 b_2]^T$, is defined where $\mathcal{U}$ is a closed and bounded set. After utilizing (6) and (7), the dynamics for $u_1(t), u_2(t)$ can be expressed as

$$\dot{u}_i = -y_3 b_3 u_i - (y_2 \omega_1 - y_1 \omega_2) u_i + q(b_i) u_i, \forall i = \{1, 2\}. \tag{8}$$

From (6) and (8) the dynamics of the known states $y_1(t), y_2(t)$ and the unknown state $\theta(t) = [y_3 \quad u_1 \quad u_2]^T$ are

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -y_1 b_3 & 1 & 0 \\ -y_2 b_3 & 0 & 1 \end{bmatrix}}_{J(y_1, y_2, b_3)} \begin{bmatrix} y_3 \\ u_1 \\ u_2 \end{bmatrix}$$
$$+ \underbrace{\begin{bmatrix} -y_1 y_2 \omega_1 + (1 + y_1^2)\omega_2 - y_2 \omega_3 \\ -(1 + y_2^2)\omega_1 + y_1 y_2 \omega_2 + y_1 \omega_3 \end{bmatrix}}_{\Psi(y_1, y_2, \omega)} \tag{9}$$

and

$$\begin{bmatrix} \dot{y}_3 \\ \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -y_3^2 b_3 - (y_2 \omega_1 - y_1 \omega_2) y_3 \\ -y_3 b_3 u_1 - (y_2 \omega_1 - y_1 \omega_2) u_1 + q(b_1) u_1 \\ -y_3 b_3 u_2 - (y_2 \omega_1 - y_1 \omega_2) u_2 + q(b_2) u_2 \end{bmatrix}}_{g(\theta, \omega, y_1, y_2, b_3)}. \tag{10}$$

Since $y_1(t)$ and $y_2(t)$ are measurable, from (1) and (4) the Euclidean structure $\bar{m}(t)$ can be estimated once the state $y_3(t)$ is determined. Since the dynamics of the outputs $y_1(t), y_2(t)$ are affine in the unknown state $\theta(t)$, a reduced order observer can be developed based on this relationship for the unknown state $\theta(t)$. The subsequent development is based on the strategy of constructing the estimates $\hat{\theta}(t) \triangleq [\hat{y}_3 \quad \hat{u}_1 \quad \hat{u}_2]^T \in \mathbb{R}^3$. To quantify the SaM estimation objective, an estimation error $\tilde{\theta}(t) = [\tilde{\theta}_1 \quad \tilde{\theta}_2 \quad \tilde{\theta}_3]^T \in \mathbb{R}^3$ is defined as

$$\tilde{\theta}(t) \triangleq [y_3 - \hat{y}_3 \quad u_1 - \hat{u}_1 \quad u_2 - \hat{u}_2]^T. \tag{11}$$

Assumption 2: The function $q(b_i, t) \forall i = \{1, 2\}$ is locally Lipschitz where $q(b_1) - q(\hat{b}_1) = \lambda_1(b_1 - \hat{b}_1)$ and $q(b_2) - q(\hat{b}_2) = \lambda_2(b_2 - \hat{b}_2)$ where λ_1 and λ_2 are Lipschitz constants.

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[2]The homography matrix can be decomposed to obtain a rotation and scaled translation of the camera between two views. The decomposition of the homography leads to two solutions, one of which is physically relevant. More details about the homography decomposition and how to obtain the physically relevant solution can be found in [21], [22].

[3]For the remainder of this technical note, the feature point subscript j is omitted to streamline the notation.

[4]An observer can be developed with any of the three linear velocities known. In this technical note, $b_3(t)$ is assumed to be known w.l.o.g.

Remark 2: The linear velocity model in (7) (and in the results in [8], [10]) is restricted to motions that are satisfied by Assumption 2; yet, various classes of trajectories satisfy this assumption (e.g., straight line trajectories, circles, some periodic trajectories, etc.).

Assumption 3: The function $J(y_1, y_2, b_3)$ defined in (9) satisfies the persistency of excitation condition, i.e., $\exists \gamma, \delta > 0 :$ $\int_t^{t+\delta} J^T(y_1(\tau), y_2(\tau), b_3(\tau)) J(y_1(\tau), y_2(\tau), b_3(\tau)) d\tau \geq \gamma I \ \forall t \geq 0$.

Remark 3: Assumption 3 is violated iff $\exists \, t_1 | \forall t > t_1, \ b_3(t) = 0$ or $y_1(t) = c_1, \ y_2(t) = c_2$. That is, Assumption 3 is valid unless there exists a time t_1 such that for all $t > t_1$ the camera translates along the projected ray of an observed feature point.

Assumption 4: The linear camera velocities $b(t)$ are upper and lower bounded by constants.

Remark 4: The following bounds can be developed using Assumption 1, Remark 1 and the definitions of $u_1(t)$ and $u_2(t)$:

$$u_{1\min} \leq u_1 \leq u_{1\max}, \ u_{2\min} \leq u_2 \leq u_{2\max}.$$

Step I: Angular Velocity Estimation: Solutions are available in literature that can be used to determine the relative angular velocity between the camera and a target [3]. To quantify the rotation mismatch between $\mathcal{F}_c^*$ and $\mathcal{F}_c$, a rotation error vector $e_\omega(t) \in \mathbb{R}^3$ is defined by the angle-axis representation as

$$e_\omega \triangleq u_\omega(t)\theta_\omega(t) \tag{12}$$

where $u_\omega(t) \in \mathbb{R}^3$ represents a unit rotation axis, and $\theta_\omega(t) \in \mathbb{R}$ denotes the rotation angle about $u_\omega(t)$ that is assumed to be confined to region $-\pi < \theta_\omega(t) < \pi$. The angle $\theta_\omega(t)$ and axis $u_\omega(t)$ can be computed using the rotation matrix $\bar{R}(t)$ obtained by decomposing the Homography matrix $H(t)$ given by the relation in (3). Taking time derivative of (12) yields

$$\dot{e}_\omega = L_\omega \omega \tag{13}$$

where $L_\omega(t) \in R^{3 \times 3}$ denotes an invertible Jacobian matrix [3]. A robust integral of the sign of the error (RISE)-based observer $\hat{e}_\omega(t) \in \mathbb{R}^3$ is generated in [3] as

$$\dot{\hat{e}}_\omega = (K_\omega + I_{3\times 3})\tilde{e}_\omega(t) + \int_{t_0}^t (K_\omega + I_{3\times 3})\tilde{e}_\omega d\tau + v$$
$$\dot{v} = \rho_\omega sgn(\tilde{e}_\omega) \tag{14}$$

where $K_\omega, \rho_\omega \in \mathbb{R}^{3 \times 3}$ are positive constant diagonal gain matrices, and $\tilde{e}_\omega(t) \in \mathbb{R}^3$ quantifies the observer error as $\tilde{e}_\omega(t) \triangleq e_\omega - \hat{e}_\omega$. A Lyapunov-based stability analysis is provided in [3] that proves

$$\dot{\hat{e}}_\omega(t) - \dot{e}_\omega(t) \to 0 \quad \text{as} \quad t \to \infty \tag{15}$$

and that all closed-loop signals are bounded. Based on (13) and (15), the angular velocity can be determined as

$$\hat{\omega}(t) = L_\omega^{-1}\dot{\hat{e}}_\omega(t) \quad \text{as} \quad t \to \infty. \tag{16}$$

An angular velocity estimation error $\tilde{\omega}(t) \in \mathbb{R}^3 \triangleq [\tilde{\omega}_1(t), \ \tilde{\omega}_2(t), \ \tilde{\omega}_3(t)]^T$ is defined as $\tilde{\omega}_i(t) = \omega_i(t) - \hat{\omega}_i(t), \forall i = \{1, 2, 3\}$. As shown in [3], the angular velocity estimator given by (14) is asymptotically stable; thus, the angular velocity estimation error $\|\tilde{\omega}(t)\| \to 0$ as $t \to \infty$.

Step II: Structure Estimation: A reduced order observer for $\theta(t)$ is designed as

$$\begin{bmatrix} \hat{y}_3 \\ \hat{u}_1 \\ \hat{u}_2 \end{bmatrix} = \begin{bmatrix} \bar{y}_3 \\ \bar{u}_1 \\ \bar{u}_2 \end{bmatrix} + \Gamma \begin{bmatrix} -\frac{b_3(y_1^2 + y_2^2)}{2} \\ y_1 \\ y_2 \end{bmatrix} \tag{17}$$

where the state vector $\begin{bmatrix} \bar{y}_3 & \bar{u}_1 & \bar{u}_2 \end{bmatrix}^T$ is updated using the following update law:

$$\begin{bmatrix} \dot{\bar{y}}_3 \\ \dot{\bar{u}}_1 \\ \dot{\bar{u}}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -\hat{y}_3^2 b_3 - (y_2\hat{\omega}_1 - y_1\hat{\omega}_2)\hat{y}_3 \\ -\hat{y}_3 b_3 \hat{u}_1 - (y_2\hat{\omega}_1 - y_1\hat{\omega}_2)\hat{u}_1 + q(\hat{b}_1)\hat{u}_1 \\ -\hat{y}_3 b_3 \hat{u}_2 - (y_2\hat{\omega}_1 - y_1\hat{\omega}_2)\hat{u}_2 + q(\hat{b}_2)\hat{u}_2 \end{bmatrix}}_{g(\hat{\theta}, \hat{\omega}, y_1, y_2, b_3)}$$
$$- \Gamma J^T \left(\begin{bmatrix} -y_1 b_3 & 1 & 0 \\ -y_2 b_3 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{y}_3 \\ \hat{u}_1 \\ \hat{u}_2 \end{bmatrix} \right.$$
$$\left. + \underbrace{\begin{bmatrix} -y_1 y_2 \hat{\omega}_1 + (1 + y_1^2)\hat{\omega}_2 - y_2\hat{\omega}_3 \\ -(1 + y_2^2)\hat{\omega}_1 + y_1 y_2 \hat{\omega}_2 + y_1\hat{\omega}_3 \end{bmatrix}}_{\Psi(y_1, y_2, \hat{\omega})} \right)$$
$$+ \Gamma \begin{bmatrix} \frac{\dot{b}_3(y_1^2 + y_2^2)}{2} \\ 0 \\ 0 \end{bmatrix}. \tag{18}$$

In (18), $\Gamma \in \mathbb{R}$, $\hat{\omega}_i(t)$ are given by (16), and $J(y, b_3)$ is defined in (9). Differentiating (11) and using (9), (10), (17) and (18) yields the following closed-loop observer error dynamics:

$$\begin{bmatrix} \dot{\tilde{\theta}}_1 \\ \dot{\tilde{\theta}}_2 \\ \dot{\tilde{\theta}}_3 \end{bmatrix} = \begin{bmatrix} (y_3 + \hat{y}_3)b_3\tilde{\theta}_1 \\ y_3 b_3 \tilde{\theta}_2 + b_3 \hat{u}_1 \tilde{\theta}_1 \\ y_3 b_3 \tilde{\theta}_3 + b_3 \hat{u}_2 \tilde{\theta}_1 \end{bmatrix}$$
$$+ (y_2\omega_1 - y_1\omega_2)\tilde{\theta}_1$$
$$+ (y_2\hat{\omega}_1 - y_1\hat{\omega}_2)\tilde{\theta}_2 + q(b_1)\tilde{\theta}_2$$
$$+ (y_2\hat{\omega}_1 - y_1\hat{\omega}_2)\tilde{\theta}_3 + q(b_2)\tilde{\theta}_3$$
$$+ (y_2\tilde{\omega}_1 - y_1\tilde{\omega}_2)\hat{y}_3$$
$$+ (y_2\tilde{\omega}_1 - y_1\tilde{\omega}_2)\hat{u}_1 + \lambda_1\left(b_1 - \hat{b}_1\right)\hat{u}_1$$
$$+ (y_2\tilde{\omega}_1 - y_1\tilde{\omega}_2)\hat{u}_2 + \lambda_2\left(b_2 - \hat{b}_2\right)\hat{u}_2$$
$$+ \Gamma J^T \left(\begin{bmatrix} -y_1 b_3 & 1 & 0 \\ -y_2 b_3 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{y}_3 \\ \hat{u}_1 \\ \hat{u}_2 \end{bmatrix} \right.$$
$$\left. + \begin{bmatrix} -y_1 y_2 \hat{\omega}_1 + (1 + y_1^2)\hat{\omega}_2 - y_2\hat{\omega}_3 \\ -(1 + y_2^2)\hat{\omega}_1 + y_1 y_2 \hat{\omega}_2 + y_1\hat{\omega}_3 \end{bmatrix} - \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} \right).$$

Using the output dynamics from (9), the error dynamics can be rewritten as

$$\dot{\tilde{\theta}} = g(\theta, \omega, y_1, y_2, b_3) - g(\hat{\theta}, \hat{\omega}, y_1, y_2, b_3)$$
$$- \Gamma J^T \left(J\tilde{\theta} + \Psi(y_1, y_2, \tilde{\omega}) \right).$$

Using Assumption 2, following relationship can be developed:

$$g(\theta, \omega, y_1, y_2, b_3) - g(\hat{\theta}, \hat{\omega}, y_1, y_2, b_3) = \Lambda\tilde{\theta} + \alpha(y_1, y_2, \hat{\theta}, \tilde{\omega}) \tag{19}$$

where

$$\Lambda(t) = \begin{bmatrix} \begin{array}{c}(y_3 + \hat{y}_3)b_3 \\ +y_2\omega_1 - y_1\omega_2\end{array} & 0 \\ b_3\hat{u}_1 + \frac{\lambda_1\hat{u}_1^2}{y_3\hat{y}_3} & \begin{array}{c}y_3b_3 + q(b_1) + \frac{\lambda_1\hat{u}_1}{y_3} \\ +y_2\hat{\omega}_1 - y_1\hat{\omega}_2\end{array} \\ b_3\hat{u}_2 + \frac{\lambda_2\hat{u}_2^2}{y_3\hat{y}_3} & 0 \\ 0 \\ 0 \\ y_3b_3 + q(b_2) + (y_2\hat{\omega}_1 - y_1\hat{\omega}_2) + \frac{\lambda_2\hat{u}_2}{y_3} \end{bmatrix} \quad (20)$$

such that $\sup_t \|\Lambda(t)\| \le \rho$, and

$$\alpha(y_1, y_2, \hat{\theta}, \tilde{\omega}) = \begin{bmatrix} (y_2\tilde{\omega}_1 - y_1\tilde{\omega}_2)\,\hat{y}_3 \\ (y_2\tilde{\omega}_1 - y_1\tilde{\omega}_2)\,\hat{u}_1 \\ (y_2\tilde{\omega}_1 - y_1\tilde{\omega}_2)\,\hat{u}_2 \end{bmatrix}. \quad (21)$$

Using (19) the error dynamics can be written as

$$\dot{\tilde{\theta}} = \Lambda\tilde{\theta} - \Gamma J^T J\tilde{\theta} - \Gamma J^T\Psi(y_1, y_2, \tilde{\omega}) + \alpha(y_1, y_2, \hat{\theta}, \tilde{\omega}). \quad (22)$$

The results from the angular velocity estimator in Section IV-A-I prove that $\tilde{\omega}_1, \tilde{\omega}_2, \tilde{\omega}_3 \to 0$ therefore, (21) can be used to conclude that $\Psi(\cdot) \to 0$ and $\alpha(\cdot) \to 0$ as $t \to \infty$.

Theorem 1: If Assumptions 1–4 are satisfied, the reduced order observer in (17) and (18) asymptotically estimates $\theta(t)$ in the sense that $\left\|\tilde{\theta}(t)\right\| \to 0$ as $t \to \infty$.

Proof: The stability of the error system in (22) can be proved using a converse Lyapunov theorem [24]. Consider the nominal system

$$\dot{\tilde{\theta}} = f(\tilde{\theta}) = -\Gamma J^T J\tilde{\theta}. \quad (23)$$

Using Theorem 2.5.1 of [25] the error system in (23) is globally exponentially stable if Assumption 3 is satisfied. Hence, $\tilde{\theta}(t)$ satisfies the inequality $\left\|\tilde{\theta}(t)\right\| \le \left\|\tilde{\theta}(t_0)\right\|\alpha_1 e^{-\alpha_2(t-t_0)}$, where $\alpha_1, \alpha_2 \in \mathbb{R}^+$, and α_2 is directly proportional to Γ and inversely proportional to δ [8], [25]. Consider a set $\mathcal{D} = \{\tilde{\theta}(t) \in \mathbb{R}^3 \mid \left\|\tilde{\theta}(t)\right\| < \infty\}$. Using a converse Lyapunov theorem there exists a function $V : [0, \infty) \times \mathcal{D} \to \mathbb{R}$ that satisfies

$$c_1 \left\|\tilde{\theta}(t)\right\|^2 \le V(t, \tilde{\theta}) \le c_2 \left\|\tilde{\theta}(t)\right\|^2,$$
$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \tilde{\theta}}(-\Gamma J^T J\tilde{\theta}) \le -c_3 \left\|\tilde{\theta}(t)\right\|^2,$$
$$\left\|\frac{\partial V}{\partial \tilde{\theta}}\right\| \le c_4 \left\|\tilde{\theta}(t)\right\| \quad (24)$$

for some positive constants c_1, c_2, c_3, c_4. Using $V(t, \tilde{\theta})$ as a Lyapunov function candidate for the perturbed system in (22), the derivative of $V(t, \tilde{\theta})$ along the trajectories of (22) is given by

$$\dot{V}(t, \tilde{\theta}) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \tilde{\theta}}(-\Gamma J^T J\tilde{\theta}) + \frac{\partial V}{\partial \tilde{\theta}}((\Lambda)\tilde{\theta}) + \frac{\partial V}{\partial \tilde{\theta}}(-\Gamma J^T\Psi(y_1, y_2, \tilde{\omega}) + \alpha(y_1, y_2, \hat{\theta}, \tilde{\omega})).$$

Using the bounds in (24) the following inequality is developed:

$$\dot{V}(t, \tilde{\theta}) \le -(c_3 - c_4\rho) \left\|\tilde{\theta}\right\|^2 + c_4 d \left\|\tilde{\theta}\right\| \quad (25)$$

where ρ is an upper bound on (20) and $d(t) = \|\Gamma\| \|J^T\| \|\Psi(y_1, y_2, \tilde{\omega})\| + \left\|\alpha(y_1, y_2, \hat{\theta}, \tilde{\omega})\right\|$
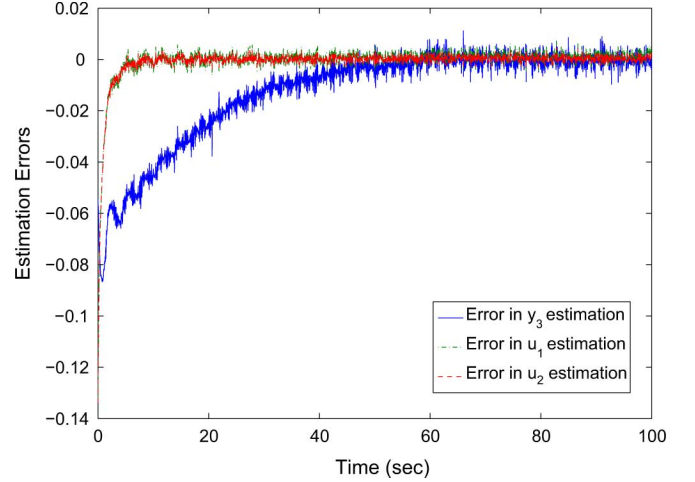

Fig. 1. State estimation error.

where $d(t) \to 0$ as $t \to \infty$. Using Theorem 4.14 of [24] the estimates of c_3 and c_4 are given by[5]

$$c_3 = \frac{1}{2}, \quad c_4 = \frac{2\alpha_1}{(\alpha_2 - L)} \left[1 - e^{-(\alpha_2 - L)\ln(2\alpha_1^2)/2\alpha_2}\right]$$

where $L \in \mathbb{R}^+$ is an upper bound on the norm of Jacobian matrix $\partial f(\tilde{\theta})/\partial\tilde{\theta}$, where $f(\tilde{\theta})$ is defined in (23). Since α_2 is directly proportional to the gain Γ, the inequality $c_3 - c_4\rho = 1/2 - 2\alpha_1\rho/(\alpha_2 - L) [1 - e^{-(\alpha_2 - L)\ln(2\alpha_1^2)/2\alpha_2}] > 0$ can be achieved by choosing the gain Γ sufficiently large. Using (24), (25), and based on the development in Section 9.3 of [24], the following bound is obtained:

$$\left\|\tilde{\theta}(t)\right\| \le \sqrt{\frac{c_2}{c_1}} e^{c_4/2c_1} \left\|\tilde{\theta}(t_0)\right\| e^{-\beta(t-t_0)} + \frac{c_4}{2c_1} e^{c_4/2c_1} \int_0^t e^{-\beta(t-\tau)} d(\tau) d\tau \quad (26)$$

where a constant convergence rate $\beta > 0$ can be increased by increasing c_3. From (26), $\left\|\tilde{\theta}(t)\right\| \in \mathcal{L}_\infty$, thus $\tilde{\theta}_1(t), \tilde{\theta}_2(t), \tilde{\theta}_3(t) \in \mathcal{L}_\infty$. Since $\tilde{\theta}_1(t), \tilde{\theta}_2(t), \tilde{\theta}_3(t) \in \mathcal{L}_\infty$, and the fact that $\theta_1(t), \theta_2(t), \theta_3(t) \in \mathcal{L}_\infty$ can be used to conclude that $\hat{\theta}_1(t), \hat{\theta}_2(t), \hat{\theta}_3(t) \in \mathcal{L}_\infty$. Using the result from Section IV-A-I that $\|\tilde{\omega}(t)\| \to 0$ as $t \to \infty$, the functions $\|\Psi(y_1, y_2, \tilde{\omega})\|, \left\|\alpha(y_1, y_2, \hat{\theta}, \tilde{\omega})\right\| \to 0$ as $t \to \infty$. Hence, $d(t) \to 0$ as $t \to \infty$ and $d(t) \in \mathcal{L}_\infty$. Since $d(t) \to 0$ as $t \to \infty$ and $d(t) \in \mathcal{L}_\infty$, by the Lebesgue dominated convergence theorem [26] $\lim_{t \to \infty} \int_0^t e^{-\beta(t-\tau)} d(\tau) d\tau = \int_0^t e^{-\beta\sigma} \lim_{t \to \infty} d(\tau - \sigma) d\sigma = \lim_{t \to \infty} d(t)/\beta = 0$ (see Theorem 3.3.2.33 of [27]). Lemma 9.6.3 of [24] can now be invoked to show that $\left\|\tilde{\theta}(t)\right\| \to 0$ as $t \to \infty$. Hence, the reduced order estimator in (17) and (18) identifies the structure of observed feature points and unknown camera motion asymptotically. Since $y_3(t)$, $u_1(t)$, and $u_2(t)$ can be estimated, the motion parameters $b_1(t)$ and $b_2(t)$ can be recovered based on the definition of $u(t)$.

V. SIMULATION

In this section, a numerical example is presented to illustrate the performance of the proposed estimator for estimating depth of a feature

[5]Note that $\lim_{\alpha_2 \to L} c_4 = \lim_{\alpha_2 \to L}\left(1 - e^{-(\alpha_2 - L)\ln(2\alpha_1^2)/2\alpha_2}/(\alpha_2 - L)\right) \ne \infty.$

point, and linear and angular velocities of the camera. The time-varying angular motion of the camera is selected as

$$\omega = \begin{bmatrix} 0.01\sin(\frac{t}{2}) & 0.01\sin(\frac{t}{2}) & 0 \end{bmatrix}^T.$$

The linear velocities along the X and Y axes are given by

$$\dot{b}(t) = \begin{bmatrix} -b_1^2(t) & -b_2^2(t) \end{bmatrix}^T$$

which conforms to (7). The linear velocity along the Z axis is selected as

$$b_3(t) = \cos(2t).$$

The initial linear velocity along the X and Y axes are selected as

$$b(t_0) = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$$

and the initial Euclidean coordinates of the first point is

$$\bar{m}(t_0) = \begin{bmatrix} 10 & 10 & 100 \end{bmatrix}^T.$$

The camera calibration matrix is selected as

$$A = \begin{bmatrix} 800 & 0 & 300 \\ 0 & 800 & 200 \\ 0 & 0 & 1 \end{bmatrix}.$$

The camera trajectory produces feature point motion in the image frame. Points are tracked in the image while the camera is moving. Image coordinates of the first point, and the camera linear velocity $b_3(t)$ are known. The states of the estimator are initialized to

$$\bar{y}_3(t_0) = 0.1, \quad \bar{u}_1(t_0) = 0.1, \quad \bar{u}_2(t_0) = 0.1.$$

The estimator gain is selected as $\Gamma = 3.6$. Measurement noise with zero mean and variance of 0.1 is added to the image point vector p defined in (4) and the linear velocity $b_3(t)$ using Matlab's "randn()" command. Asymptotic convergence of the estimation error is shown in Fig. 1 in the presence of noisy measurement inputs.

VI. Conclusion

A reduced order observer is developed for the estimation of the structure (i.e., range to the target and Euclidean coordinates of the feature points) of a stationary target with respect to a moving camera, along with two unknown time-varying linear velocities and the angular velocity. The angular velocity is estimated using Homography relationships between two camera views. The observer requires the image coordinates of the points, a single linear camera velocity, and the corresponding linear camera acceleration in any one of the three camera coordinate axes. Under a physically motivated PE condition, asymptotic convergence of the observer is guaranteed. Having at least some partial motion knowledge and a similar observability condition to the given PE condition are likely necessary in future solutions to the SaM problem. However, future efforts could potentially eliminate the need for any model of the vehicle trajectory (even if uncertain as in this result) and eliminate the need for an acceleration measurement.

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